NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES

SCHOOL OF NATURAL AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science; Bachelor of Science in Applied Mathematics and Statistics			
QUALIFICATION CODE:	07BSOC; 07BSAM	**LEVEL:**	5
COURSE CODE:	LIA502S	**COURSE CODE:**	LINEAR ALGEBRA 1
SESSION:	JULY 2023	**PAPER:**	THEORY
DURATION:	3 HOURS	**MARKS:**	100

SUPPLEMENTARY / SECOND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER:	DR. DSI IIYAMBO
MODERATOR:	DR. N CHERE

INSTRUCTIONS

1. Attempt all the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in black or blue inked, and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1

Consider the vectors $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

a) Find the angle θ (in radians) that is between $\mathbf{a}$ and $\mathbf{b}$. [5]

b) Find a unit vector that is perpendicular to both vectors $\mathbf{a}$ and $\mathbf{b}$. [7]

Question 2

Consider the following matrices.

$$A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & 1 \\ -1 & 3 \\ 2 & -2 \end{pmatrix}, \quad \text{and } D = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 2 & 1 \end{pmatrix}.$$

a) Given that $C = AB$, determine the element c_{32}. [3]

b) Find $(3A)^T$. [3]

c) Is DB defined? If yes, then find it, and hence calculate $tr(DB)$. [6]

Question 3

Let A be a square matrix.

a) What does it mean to say that A is a skew-symmetric matrix? [2]

b) Prove that $A - A^T$ is a skew-symmetric matrix. [5]

c) Prove that AA^T is a symmetric matrix. [4]

Question 4

Consider the matrix $B = \begin{pmatrix} 1 & 2 & 1 \\ 3 & -2 & -4 \\ 2 & 3 & -1 \end{pmatrix}$.

a) Use the *Cofactor expansion method*, expanding along the first row, to evaluate the determinant of B. [8]

b) Is B invertible? If it is, use Gaussian reduction to find B^{-1}. [14]

c) Find $\det\left(((2B)^{-1})^T\right)$. [6]

Question 5

Use *Cramer's Rule* to find the solution of the following system of linear equations, if it exists.

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 6 \\ x_1 + 2x_2 + 4x_3 &= 9 \\ 2x_1 + x_2 + 6x_3 &= 11 \end{aligned}$$

[8]

Question 6

a) Prove that in a vector space, the negative of a vector is unique. [9]

b) Determine whether the following set is a subspace of $\mathbb{R}^n$.

$$S = \{(a, b, c) \in \mathbb{R}^n \mid a + b + c = 0\}$$

[13]
